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Love Waves in Layered Graded Composite Structures with Imperfectly Bonded Interface

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Abstract

Theoretical analysis and numerical calculations of Love wave propagation in layered graded composites with imperfectly bonded interface are described in this paper. On the basis of WKB method, the approximate analytic solutions for Love waves are obtained. By the interface shear spring model, the dispersion relations for Love waves in layered graded composite structures with rigid, slip, and imperfectly bonded interfaces are given, and the effects of the interface conditions on the phase velocities of Love waves in SiC/Al layered graded composites are discussed. Numerical analysis shows that the phase velocity decreases when the defined flexibility parameter is greater. For the general imperfectly bonded interface, the phase velocity changes in the range of the velocities for the rigid and slip interface conditions.

Keywords: Love wave; layered graded composites; WKB method; dispersion relation

Functionally graded materials (FGMs) were first investigated by Japanese scientists in 1984. They found that FGMs are superheat-resistive, and have attractive applications in space structures. Now FGMs are widely used in fusion reactors, biomedical, electronic fields and are promising candidates for future advanced technological applications. FGMs exhibit a continuous variation in chemical compositions or microstructural parameters over geometrical distance. It is this continuous change that enhances the bonding strength, reduces the residual stresses, improves the strength of the composite structure significantly, and makes the FGM different in behavior from the traditional laminated composites[1]. In recent years, the mechanical behaviors of these materials and structures have attracted considerable attention. Much of the attention has been on the crack problems in FGMs, and few insights exist concerning the wave propagation problems in FGMs. Two fundamental approaches are often applied to deal with the elastic wave problems in FGMs, i.e., homogeneous models and inhomogeneous models. For the homogeneous model, the material is divided into arbitrary numbers of homogenous layers and each layer has constant physical characteristics. For the inhomogeneous model, the material properties change continuously in the thickness direction. Due to the complexity of the governing equations, the perturbation method, the WKB approximation method, and other kinds of approximation methods are applied. In respect that the solutions of the WKB method are relatively simple and the dispersion relations can be obtained, in this paper this method is used to study Love waves in layered graded composite structures.

In general, layered composites are assumed to be perfectly bonded, i.e., the rigid interface condi-

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tion. Although, in practice, loading or manufacture process may lead to the presence of defects and cracks in the interface, and the interface may be unable to transfer stresses perfectly because of the displacement discontinuity, which is called as an imperfect interface[2]. Linear spring model is generally used to simulate the imperfect interface, because this model is capable to accommodate both rigid interface condition and fully slipping interface condition. For a slip interface, no shear stiffness and strength but the normal stiffness is same with the adjacent two solid media. The effects of the imperfect interface on elastic waves were widely studied, for example, the effects of the imperfect interface on Rayleigh waves and Lamb waves were respectively studied by the interface spring models in Refs.[3-4], and also through the measurements of these waves, the interface properties were evaluated[5-6]. Recently waves in layered graded materials are of great interest[7-9]. Most of them are concerned with waves in composites with perfectly bonded interface. In this paper, based on the WKB method, the Love wave propagation in layered graded composite structures with imperfect interfaces is investigated by the shear spring model, and the effects of the imperfect interface on Love waves are discussed.

1 Statement of the Problems

The layered graded composite structure and the corresponding coordinate system are illustrated in Fig.1. It is assumed that the coating is in the region $-h<x_1<0$ and the substrate $0<x_1<\hat{h}$. The upper surface $x_1=-h$ is traction free. The coating is a graded material and the material parameters vary smoothly in the thickness direction. The substrate is a homogenous isotropic material. In general, the thickness of the substrate is much greater than that of the coating and the substrate can be treated as a half space. If the bulk shear velocities of the coating and substrate are denoted as c_{s0} and c_{s1}, respectively, the phase velocity c of Love wave changes in the range $c_{s0}<c<c_{s1}$.

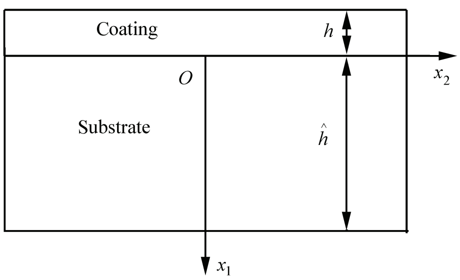

Fig.1 Layered graded composite structure.

For Love wave propagation with phase velocity c along the x_2 axis, the mechanical displacement components u_i are as follows

$$\begin{aligned} u_1 &= u_2 = 0 \\ u_3 &= u_3(x_1, x_2, t) \end{aligned} \tag{1}$$

The stress equation of motion for the coating is

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} = \rho \frac{\partial^2 u_3}{\partial t^2} \tag{2}$$

where σ_{31} and σ_{32} are stress components, and ρ is density. For the generally used FGMs, the Poisson's ratio and density are assumed to be space independent, and only the elastic constants change in the thickness direction. Assume that the shear modulus $\mu = \mu_0 f(x_1)$, here μ_0 is the shear modulus of the coating at surface $x_1=-h$, f is a function of x_1. The constitutive equations for the coating are

$$\sigma_{31} = \mu_0 f(x_1) \frac{\partial u_3}{\partial x_1}, \quad \sigma_{32} = \mu_0 f(x_1) \frac{\partial u_3}{\partial x_2} \tag{3}$$

From Eqs.(2) and (3), the equation of motion for the coating can be rewritten as

$$\frac{\partial^2 u_3}{\partial x_1^2} + \frac{f'}{f} \frac{\partial u_3}{\partial x_1} + \frac{\partial^2 u_3}{\partial x_2^2} = \frac{\ddot{u}_3}{c_{s0}^2 f} \tag{4}$$

where the double dot denotes time differentiation, the superscripts "′" denotes differentiation with respect to x_1, i.e., $f' = \mathrm{d}f/\mathrm{d}x_1$ and $c_{s0} = \sqrt{\mu_0/\rho}$. The equation of motion for the substrate is

$$\frac{\partial^2 \hat{u}_3}{\partial x_1^2} + \frac{\partial^2 \hat{u}_3}{\partial x_2^2} = \frac{\ddot{\hat{u}}_3}{c_{s1}^2} \tag{5}$$

where $c_{s1} = \sqrt{\mu_1/\rho_1}$, μ_1, ρ_1 and $\hat{u}_3$ are the shear modulus, density, and displacement of the substrate, respectively.

Assume the solutions of the coating and the substrate as

$$u_3(x_1, x_2, t) = W(x_1)\exp[\mathrm{i}\kappa(x_2 - ct)] \tag{6}$$

$$\hat{u}_3(x_1, x_2, t) = \hat{W}(x_1)\exp[\mathrm{i}\kappa(x_2 - ct)] \tag{7}$$

where κ is wave number, c is phase velocity, i= $\sqrt{-1}$, $W(x_1)$ and $\hat{W}(x_1)$ are unknown functions.

2 Solutions of the Problems

Substituting Eq.(7) into Eq.(5), one can obtains

$$\hat{W}'' + \left[\frac{c^2}{c_{s1}^2} - 1\right]\kappa^2 \hat{W} = 0 \tag{8}$$

Let $r_1 = \sqrt{1 - c^2 / c_{s1}^2}$, considering the condition that as $x_1 \to \infty$, the displacement $\hat{u}_3 \to 0$, thus the solution to Eq.(8) is

$$\hat{W} = C_3 e^{-\kappa r_1 x_1} \tag{9}$$

where C_3 is an unknown constant.

Substituting Eq.(6) into Eq.(4), yields

$$W'' + \frac{f'}{f}W' + \left(\frac{c^2}{c_{s0}^2 f} - 1\right)\kappa^2 W = 0 \tag{10}$$

According to the WKB method[7,10], the approximate solution to Eq.(10) is

$$W(x_1) = \exp[\int \phi(x_1)\mathrm{d}x_1] \tag{11}$$

where

$$\phi = \kappa\phi_0 + \phi_1 + \kappa^{-1}\phi_2 + \kappa^{-2}\phi_3 + \kappa^{-3}\phi_4 + \cdots \tag{12}$$

Substitution of Eq.(11) into Eq.(10), yields

$$\phi' + \phi^2 + \frac{f'}{f}\phi + \left(\frac{c^2}{c_{s0}^2 f} - 1\right)\kappa^2 = 0 \tag{13}$$

Inserting Eq.(12) into Eq.(13), and equating the coefficients of the same order of κ, leads to

$$\left.\begin{aligned} &\phi_0 + \frac{c^2}{c_{s0}^2 f} - 1 = 0 \\ &\phi_0' + 2\phi_0\phi_1 + \frac{f'}{f}\phi_0 = 0 \\ &\phi_1' + 2\phi_0\phi_2 + \phi_1^2 + \frac{f'}{f}\phi_1 = 0 \\ &\cdots \end{aligned}\right\} \tag{14}$$

Generally for the crack problems in FGMs, the material constants are assumed to vary as an exponential or power function of the thickness direction. Here it is assumed that the shear modulus of the coating is a power function of the x_1 axis, i.e.,

$$f(x_1) = [1 + \eta(h + x_1)]^2 \tag{15}$$

where η is the gradient constant.

If $\phi = \kappa\phi_0 + \phi_1$, inserting Eq.(15) into Eq.(14), results in

$$\left.\begin{aligned} \phi_0 &= \frac{\sqrt{c_{s0}^2 g^2 - c^2}}{c_{s0} g} \\ \phi_1 &= -\frac{\eta\left(2c_{s0}^2 g^2 - c^2\right)}{2g\left(c_{s0}^2 g^2 - c^2\right)} \end{aligned}\right\} \tag{16}$$

where $g = 1 + \eta(h + x_1)$.

Substituting Eq.(16) into Eq.(11), yields

$$W(x_1) = \exp\left[\int \frac{\kappa\sqrt{c_{s0}^2 g^2 - c^2}}{c_{s0} g} - \frac{\eta\left(2c_{s0}^2 g^2 - c^2\right)}{2g\left(c_{s0}^2 g^2 - c^2\right)}\mathrm{d}x_1\right] \tag{17}$$

Because the existence conditions of Love waves are $c_{s0} < c_{s0}(1+\eta h) < c < c_{s1}$, let $\cot^2\gamma = c_{s0}^2 g^2 - c^2$, then $\cot\gamma = \pm \mathrm{i}r$, $r = \sqrt{c^2 - c_{s0}^2 g^2}$. Thus the displacements of the coating and the substrate are

$$\begin{aligned} u_3(x_1, x_2, t) = &\frac{1}{\sqrt{g}\left(c^2 - c_{s0}^2 g^2\right)^{1/4}} \bullet \\ &\left\{C_1 \exp\left[\frac{\mathrm{i}\kappa}{c_{s0}\eta}\left(r + c\ln\frac{c-r}{c_{s0} g}\right)\right] + \right. \\ &\left. C_2 \exp\left[-\frac{\mathrm{i}\kappa}{c_{s0}\eta}\left(r + c\ln\frac{c-r}{c_{s0} g}\right)\right]\right\} \bullet \\ &\exp[\mathrm{i}\kappa(x_2 - ct)] \end{aligned} \tag{18}$$

and

$$\hat{u}_3(x_1, x_2, t) = C_3 e^{-\kappa r_1 x_1} \exp[\mathrm{i}\kappa(x_2 - ct)] \tag{19}$$

where C_1 and C_2 are unknown constants.

3 Solutions of the Phase Velocity of Love Waves

From the traction free condition at $x_1 = -h$, one can obtain

$$\sigma_{31}(-h, x_2) = 0 \tag{20}$$

For the rigid interface case, the components of the displacement and stress components σ_{1j} (j = 1,2,3) of the two adjacent media are continuous along the interface, i.e., the continuity conditions at the interface $x_1 = 0$ are

$$\left.\begin{aligned} u_3(0, x_2) &= \hat{u}_3(0, x_2) \\ \sigma_{31}(0, x_2) &= \hat{\sigma}_{31}(0, x_2) \end{aligned}\right\} \tag{21}$$

For the slip case, the boundary conditions at the interface $x_1 = 0$ are

$$\sigma_{31}(0, x_2) = \hat{\sigma}_{31}(0, x_2) = 0 \tag{22}$$

For the imperfect interface, the boundary conditions at the interface x_1=0 can be written from the interface shear spring model,

$$\left.\begin{aligned} &u_3(0,x_2)-\hat{u}_3(0,x_2)=-R\hat{\sigma}_{31}(0,x_2) \\ &\sigma_{31}(0,x_2)=\hat{\sigma}_{31}(0,x_2) \end{aligned}\right\} \tag{23}$$

where R is the flexibility parameter.

Inserting Eqs.(18) and (19), and corresponding stress components into the boundary conditions (20) and (23), one can obtain a system of three homogenous equations in the unknowns C_1, C_2 and C_3 of the layered graded composite structures with imperfectly bonded interface,

$$\left.\begin{aligned} &d\mathrm{e}^{\mathrm{i}p}C_1+d\mathrm{e}^{-\mathrm{i}p}C_2-\left(R\mu_1\kappa r_1+1\right)C_3=0 \\ &\mu_0\left(1+\eta h\right)^2 d\left[\left(v+\mathrm{i}q\right)\mathrm{e}^{\mathrm{i}p}C_1+\left(v-\mathrm{i}q\right)\mathrm{e}^{-\mathrm{i}p}C_2\right]+ \\ &\qquad \mu_1\kappa r_1 C_3=0 \\ &\left(s+\mathrm{i}n\right)\mathrm{e}^{\mathrm{i}m}C_1+\left(s-\mathrm{i}n\right)\mathrm{e}^{-\mathrm{i}m}C_2=0 \end{aligned}\right\} \tag{24}$$

where

$$d=\frac{1}{\sqrt{1+\eta h}\left[c^2-c_{\mathrm{s}0}^2\left(1+\eta h\right)^2\right]^{1/4}} \tag{25}$$

$$n=\frac{\kappa\sqrt{c^2-c_{\mathrm{s}0}^2}}{c_{\mathrm{s}0}} \tag{26}$$

$$p=\frac{\kappa}{c_{\mathrm{s}0}\eta}\left[\sqrt{c^2-c_{\mathrm{s}0}^2\left(1+\eta h\right)^2}+ c\ln\frac{c-\sqrt{c^2-c_{\mathrm{s}0}^2\left(1+\eta h\right)^2}}{c_{\mathrm{s}0}\left(1+\eta h\right)}\right] \tag{27}$$

$$v=\frac{\eta}{2}\left[\frac{c_{\mathrm{s}0}^2\left(1+\eta h\right)}{c^2-c_{\mathrm{s}0}^2\left(1+\eta h\right)^2}-\frac{1}{1+\eta h}\right] \tag{28}$$

$$q=\frac{\kappa\sqrt{c^2-c_{\mathrm{s}0}^2\left(1+\eta h\right)^2}}{c_{\mathrm{s}0}\left(1+\eta h\right)} \tag{29}$$

$$s=\frac{\eta}{2}\left(\frac{c_{\mathrm{s}0}^2}{c^2-c_{\mathrm{s}0}^2}-1\right) \tag{30}$$

$$m=\frac{\kappa}{c_{\mathrm{s}0}\eta}\left[\sqrt{c^2-c_{\mathrm{s}0}^2}+c\ln\frac{c-\sqrt{c^2-c_{\mathrm{s}0}^2}}{c_{\mathrm{s}0}}\right] \tag{31}$$

The dispersion equation can be obtained by imposing the nontrivial solution condition of Eq.(24), i.e.,

$$\tan(p-m)=\frac{n\left[\mu_0(1+\eta h)^2 v+\dfrac{\mu_1 kr_1}{1+R\mu_1 kr_1}\right]-\mu_0(1+\eta h)^2 qs}{s\left[\mu_0(1+\eta h)^2 v+\dfrac{\mu_1 kr_1}{1+R\mu_1 kr_1}\right]+\mu_0(1+\eta h)^2 qn} \tag{32}$$

It is noted that the dispersion relations for the perfectly bonded and slip interfaces can be obtained directly from the dispersions relation for the imperfect condition. As $R=0$ and $R\to\infty$, the dispersion relations for a perfectly bonded interface and slip interface cases can be given from Eq.(32), respectively.

4 Discussions

As an example, numerical calculations are performed for the SiC/Al graded coating and Al substrate. The material constants are as follows:

SiC/Al: $\mu_0 = 29.5$ GPa , $\rho_0 = 3\ 200$ kg/m^3

Al: $\mu_1 = 26.3$ GPa, $\rho_1 = 2\ 700$ kg/m^3

Thus the bulk shear velocities of the coating and the substrate are $c_{\mathrm{s}0} = 3\ 036.2$ m/s and $c_{\mathrm{s}1} =$ 3 121 m/s, respectively. Assume that the gradient constant $\eta = 0.1$ and the thickness of the coating h = 5 mm. In Fig.2, the relations between the dimensionless phase velocity $c/c_{\mathrm{s}0}$ and the product of the wave number and coating thickness κh are plotted. The three lines in Fig.2 correspond to the rigid interface, the imperfect interface (R = 1×10^{-12}), and the slip interface conditions, respectively.

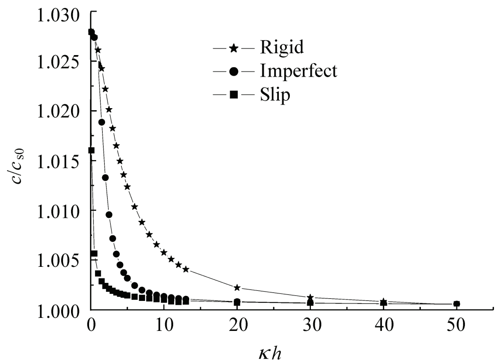

Fig.2 The dispersion relations for the fundamental Love wave mode of a graded composite structure.

It is seen from Fig.2 that as $\kappa h\to 0$, the thickness of the coating is very small compared to the wavelength, the phase velocity of Love wave approaches to the bulk shear velocity of the substrate. As $\kappa h\to\infty$, the thickness of the coating is very large compared to the wavelength, and the phase velocity of Love wave approaches to the bulk shear velocity of the coating. The interface has little effects on the phase velocities for the two cases. However for other frequencies, the imperfect interface has great effects on the phase velocities of Love waves. For the rigid interface condition, the phase velocity decreases slowly as κh increases. For the imperfect interface case, the phase velocity decreases and tends to the bulk shear velocity of the coating rapidly as κh increases. For a given value of κh, the weaker of the interface (i.e., the larger of R), the lower of the wave speed. Because the slip interface condition is an extreme condition of the imperfect case, the phase velocity for the imperfect condition ranges from the velocity for the slip interface case to the velocity for the perfectly bonded condition.

5 Conclusions

Based on the shear spring model and from the basic equations for the layered graded composite structures, the dispersion equations for Love wave propagation in layered graded composites structures are derived, the dispersion relations are shown for the rigid, imperfect, and slip interface cases. From the numerical results it is found that the interface has great effects on the dispersion relations of Love waves. Thus it is capable to determine the interface characteristics of the layered composite structures by the measurements of Love wave speeds, and the theory is promising for nondestructive characterizations of interfacial properties of the layered graded composite structures.

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